J. Fluid Mech. (1996), *vol.* 310, *pp.* 375–378

REVIEWS

Exponential Attractors for Dissipative Evolution Equations. By A. EDEN, C. FOIAS, B. NICOLAENKO and R. TEMAM. Wiley, 1994. 182 pp. £24.95.

Nonstandard Methods for Stochastic Fluid Mechanics. By M. CAPIŃSKI and N. J. CUTLAND. World Scientific, 1995. 227 pp. £42.

The book by Eden *et al.*, as its title suggests, is concerned with demonstrating that various PDEs have certain global properties. The background is the following: a *global* attractor for a PDE is a compact, connected set in some function space that absorbs all orbits (solutions) starting from all smooth initial conditions. Consequently, it contains all the long-time dynamics of solutions and, in approximate terms, is a way of looking at PDEs in this category as infinite-dimensional dynamical systems. What connects this with computations, for example, is that one can often estimate the fractal dimension of this global attractor and interpret this as the number of degrees of freedom in the system in Landau's sense. In turn, this will give an estimate of the resolution that may be needed in a numerical integration. The two-dimensional Navier-Stokes, Ginzburg–Landau, various reaction diffusion, and the Kuramoto–Sivashinsky equations all have this property although the question is still open for the three-dimensional Navier–Stokes equations. Estimates for two-dimensional Navier–Stokes are so good (Constantin, Foias & Temam 1988) that to within logarithmic corrections, the rigorous theory reproduces the Kraichnan length as the natural length scale for flows starting from all smooth initial conditions. In a more restricted set of examples, the existence of an *inertial manifold* can be demonstrated, on which solutions of the PDE in question can be shown to be completely determined by a finite set of Fourier modes. High modes are slaved to low modes on this manifold. The present book concerns itself with an object which, roughly speaking, lies halfway between a global attractor and an inertial manifold, called an *exponential attractor*. The book includes a nice introduction giving some history and background to these topics, although the referencing is slightly eccentric. After explaining the idea of exponential attractors for maps and then first-order dissipative equations, later chapters then go on to show how to approximate them. Various examples of where they apply are given and then a chapter on second-order equations is presented. The book, residing in a research note series, is well written and suitable for those who have studied the earlier work on global attractors (Constantin & Foias 1988; Temam 1988) and have a grasp of the necessary techniques such as applied functional analysis and PDEs.

The book by Capiński & Cutland has two major themes. The first is to use non-standard analysis to reproduce the results of standard Navier–Stokes analysis such as existence, uniqueness, and construction of weak solutions. Non-standard analysis uses an extended number system which includes both infinitesimal and infinite quantities. Standard notions of convergence are thereby modified, and in the end one projects down to the standard number system to recover familiar results. Non-standard analysis arguably provides neater and more compact derivations of results, and the authors make this case in their introduction to non-standard methods. The second main part of the book is about statistical solutions of the Navier–Stokes equations with stochastic forcing in the sense of Hopf (1952) and Foias (1973). One can specify random initial conditions in terms of a distribution and then attempt to find the evolution of the ensemble or, different again, one can randomly force the Navier–Stokes equations and look at the statistics of the solution. The authors do both in separate chapters, using methods of non-standard analysis throughout. The final chapter is on the Euler equations, using many of the previously developed ideas. The book is a complete compendium of notions of non-standard analysis results and methods. The necessary background to read it would be functional analysis, PDEs and stochastic differential equations. It is an *analysis* book although the authors have made a laudable effort to motivate the analysis.

REFERENCES

CONSTANTIN, P. & FOIAS, C. 1988 *Navier–Stokes Equations*. The University of Chicago Press.

CONSTANTIN, P., FOIAS, C. & TEMAM, R. 1988 On the dimension of the attractors in two dimensional turbulence. *Physica* D **30**, 284–296.

FOIAS, C. 1973 Statistical study of Navier–Stokes equations I. *Rend. Sem. Mat. Univ. Padova* **48**, 219–348.

HOPF, E. 1952 Statistical hydrodynamics and functional analysis. *J. Rat. Mech. Anal.* **1**, 87–123.

TEMAM, R. 1988 *Infinite Dimensional Dynamical Systems in Mechanics and Physics*. Springer.

J. D. GIBBON

Fluid Mechanics for Petroleum Engineers. By E. BOBOK. Elsevier, 1993. 400 pp. ISBN 0 444 98668 5. $187.50.

Mathematical Theory of Oil and Gas Recovery. By P. BEDRIKOVETSKY. Kluwer, 1993. 575 pp. ISBN 0 7923 23815. Dfl 325 or $199 or £129.

The first book reviewed here is number 32 in Elsevier's Developments in Petroleum Science Series, most of which texts are extremely specific to petroleum engineering. Bobok's textbook is however a straightforward and traditional one in engineering fluid mechanics. Most of it is given over to what the cover describes as a 'balanced treatment of the fundamental aspects of fluid flow' and as such it joins a host of similar books published over the past 50 years, usually based on the authors' lectures to students and covering rather old material. In some the author concerned had introduced a large enough number of new ideas, results or interpretations to justify replacement of older texts by the new one.

There is no strong reason that I can find for doing so in the case of this particular book. The two chapters that are of specific relevance and importance to petroleum engineering, on non-Newtonian and multi-phase flow, are rather cursorily written, with little application to specific oilfield problems. One glaring omission is flow in porous media, which is the topic that dominates reservoir engineering, and in which significant progress has been made in recent years. Perhaps this will form the subject matter for a later text in the series; for the present, the topic is partly covered by Lake's *Fundamentals of Reservoir Engineering*.

The second text reviewed here is very much about flow in porous media. By restricting attention to thin layered reservoirs, the author is able to represent the relevant flows by simple analytic models; he shows that the nature of the solutions of the relevant initial-value problems is dominated by the hyperbolic character of the models, and shows how graphical representation of the resulting moving fronts can be obtained. Most of the results are for unidirectional plane flow, but their extension to reservoir flows for injector/producer well systems is explained, as are gravitational effects giving vertical velocity components. A very large range of physical processes is described in the context of a number of ex-USSR oil and gas fields.

The author starts with one-dimensional motion of a two-phase system of immiscible liquids, and rapidly reaches the graphical/analytical solution of the well-known Buckley–Leverett problem with its discontinuous saturation profile. Everything follows naturally from this standard but powerful approach. There is a brief chapter relating percolation models to relative phase permeabilities, indicating how mercury porosimetry can be used to provide the necessary continuum functions. The relative importance of capillary, gravity and dispersion effects compared with the imposed flooding flow is explained in terms of dimensionless groups and so the meaning of a thin layered reservoir is given analytic form. Fluid compressibility is then introduced to complete this first (water-flooding) part of the book.

The second part concerns chemical flooding, and covers the partitioning of the chemical, dissolved in the injected water phase, between the relevant phases (not all of which will be relevant in every case): the aqueous, the oleic, the oil–water interface and the rock surface. Both equilibrium and non-equilibrium models are presented. The hyperbolic systems are of higher order than those governing simple flooding, thus developing multiple fronts, and have a lot in common with those developed to describe chromatographic processes. Reference is made to obtaining, from observations, the dimensional parameters or functions used in the models.

Part III covers hot-water flooding, a technique necessary when waxy crudes are being produced. Part IV covers gas and solvent injection into condensate and oil reservoirs, and includes the water–alternate–gas process; in effect this introduces three-phase flowing systems with their added complications. Part V is relatively short, covering *in-situ* gas sweetening achieved by flowing sour gas through an iron-bearing reservoir. In all of these first five parts, which represent the bulk of the book, the same style of solution is used, involving moving wave fronts in basically parallel flow. Gravitational and capillary/gravitational stratification, and convective instability are dealt with in the last two parts.

The text is not for beginners. The relevant theory of hyperbolic differential equations is presented briefly and succinctly, as are the graphical constructions used throughout. For those familiar with some of the simpler models, this allows an immense amount of information to be provided in a book of under 600 pages, covering as it does much Russian and FSU work that is not generally well-known in the West. However it could not be used as an undergraduate or graduate course text, though it would be a great help to the teacher and an excellent reference book for the aspiring research worker or the practising reservoir engineer. The translation has been sponsored by British Gas, Agip, BP, Shell and Chevron; presumably they regard it as an essential addition to the shelves of their reservoir engineers.

I found several errors in the text and was disappointed with much of the type setting and notation of figures, but these are details barely affecting the strong and consistent approach provided by this latest exponent of the Russian school of reservoir modelling. He pays generous tribute to his teachers, co-workers and students, and gives numerous references to other work in what is really an extended research monograph. What is most revealing is the total lack of reference to computational (numerical) techniques or to geostatistics and Monte Carlo methods: understanding the basic characteristics of a reservoir, to determine the most appropriate forms of secondary and tertiary recovery, has apparently dominated the FSU's thinking, perhaps because they lacked powerful computer technology. This book will suggest an alternative approach to the West's preoccupation with computer simulations which, if wrongly used, provide a vast amount of irrelevant and confusing detail and so inhibit clear and justified decision making in reservoir exploitation.

J. R. A. Pearson

A One-dimensional Introduction to Continuum Mechanics. By A. J. ROBERTS. World Scientific, 1994. 162 pp. ISBN 981 02 1913 X. $24.

Either derived from, or as a plan for, a thirty-two lecture undergraduate course this book provides an introduction to continuum mechanics. The vehicle for this is the dynamics of a one-dimensional continuum. A 'flow chart' in the preface reveals the author's *modus operandi.* The successive chapters each provide a backbone to the main theme, with all but the first containing illustrative side branches. That first chapter concentrates upon descriptions of the motion of a continuum, Lagrangian and Eulerian, and introduces the material derivative. The next two concentrate upon conservation of mass and momentum in one dimension. With the continuity equation established, side branches describe traffic flow, with the opportunity to solve first-order partial differential equations and introduce shock-wave concepts, and the aggregation of slime mold amoebae. The momentum equation includes a normal stress term, which as a side branch allows a discussion of the adiabatic flow of an ideal gas, in particular wave propagation in such a gas. The notion of strain and the introduction of constitutive relations follow naturally in the next chapter, with particular attention devoted to the elastic bar, and viscoelastic materials. The Newtonian fluid makes only a brief, and slightly apologetic, appearance since for most real fluids, 'their stress–strain relation is nonlinear and also depends upon the previous history of the fluid'. This appears to deny the outstanding success of Newtonian fluid dynamics, as evidenced by the contents of this Journal over a forty-year period. The penultimate chapter introduces quasi-one-dimensional flows, with side branches of arterial blood flow and hydraulics, whilst a final brief chapter introduces further applications of one-dimensional continuum mechanics. The book achieves its stated aims in the sense that students will be encouraged, from the interesting range of phenomena presented, to pursue the subject further, and will not be daunted when they meet key concepts within a three-dimensional framework.

N. RILEY